

Nordic Systems Copper
Facade, Roof and Rainwater
Structures

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LUVATA
Partnerships beyond metals



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Copper Characteristics for Design

Material properties

The copper used for roofs and façades is primarily of the phosphorus deoxidised copper type designated Cu-DHP in accordance with EN 1172 (European standard, Copper and copper alloys. Sheet and strip for building purposes). Table 1 on page 4 shows values for copper sheeting and strips designed for construction purposes.

Cu-DHP is a metal with good properties for standard forming and working.

There is a direct relationship between tensile strength and elongation:

- the lower the tensile strength, the greater the elongation
- the greater the elongation, the higher the malleability

The tensile strength of copper can be increased through repeated forming, and it can be lowered again by heating (annealing).

In its soft state, Cu-DHP R220, copper has the highest elongation of all construction metals. This property makes it especially suitable for complicated constructions that require a high degree of shaping. Cu-DHP R240 (half hard temper) is usually used for pitched and flat roofing, wall cladding and roof drainage systems. Cu-DHP R290 (hard temper) is recommended for the manufacture of cassettes and profiled sheeting.

Forming of copper

Copper has the highest ultimate elongation of all structural metals, and is thus particularly suitable for forming complex shapes. The hot-workability of this metal is very good. The recommended temperature is 750 °C - 900 °C. Low temperature does not influence the malleability of copper. It is not brittle when cold and it is workable at any temperature without requiring special measures to be taken.

Copper's high melting point of 1083 °C enables it to be employed in any seam connection technique. Its high melting point makes copper extremely resistant to heat, which allows it to effectively prevent the spread of fire. The following connection methods may be used:

Soft soldering

- Tin-copper or lead-tin soft solder
- Working temperature max. 450 °C
- Use manufacture's guidance

Hard soldering

- Copper-silver-phosphor or copper-phosphor solder
- Working temperature above 450 °C
- Use manufacture's guidance

Riveting, screwing and nailing

Mechanical joints in copper are made using connectors made of copper or stainless steel, and the splints for rivet are made of stainless steel or bronze. Riveting with additional soldering ensures a better joint.

Welding

Copper can be welded. However, its high thermal conductivity can make it more difficult to heat up the joint as the heat is easily dissipated from this point, particularly in the case of rough pieces.

Gas welding	→	easy
Metal arc welding	→	difficult
Gas-shielded arc welding	→	very easy
Spot and seam welding	→	possible at metal thickness of less than 1.5 mm.
Butt welding	→	easy
Flash welding	→	possible
Braze welding	→	very easy

Thermal expansion and buckling in design

When designing and manufacturing copper structures, it is important to take into account movements and forces – in both the roofing and façade materials and between the various building materials used – which manifest themselves during temperature variations.

Façade claddings and roof coverings may be subject to major temperature fluctuations both during single 24-hour periods and over the course of a calendar year. It is important to be aware of this to be able to design movement joints and details for roof coverings and wall claddings in the right way.

Mechanical properties EN-1172

Table 1

Designations		Material condition	Tensile strength		0.2% proof strength		Elongation	Hardness	
Material	Number		R_m N/mm ²		$R_{p0.2}$ N/mm ²		A_{50mm} %	HV	
Symbol	Number		min.	max.	min.	max.	min.	min.	max.
Cu-DHP CuZn0.5	CW024A CW119C	R220	220	260	-	140	33	-	-
		H040	-	-	-	-	-	40	65
		R240	240	300	180	-	8	-	-
		H065	-	-	-	-	-	65	95
		R290	290	-	250	-	-	-	-
		H090	-	-	-	-	-	-	90

NOTE: 1 N/mm² is equivalent to 1 MPa

Designations

Table 2

	Denomination	Number of Standard	Material designation	Material condition designation	Nominal dimensions in millimetres
Sheet	EN 1172	-	- Cu-DHP	- R240	- 0.6 x 1000 x 2000
or Sheet	EN 1172	-	- CW024A	- R240	- 0.6 x 1000 x 2000

Dimensions

Table 3

Surface	Dimensions	
	Thickness	Width
Nordic Standard (plain copper)	0.5 ... 5.0 mm	up to 1050 mm
Nordic Brown™	0.5 ... 1.5 mm	up to 1050 mm
Nordic Green PLUS™	0.5 ... 1.5 mm	up to 1050 mm

Physical properties of Cu-DHP

Table 4

Density	8.9 g/m ³
Melting point	1083 °C
Specific thermal capacity at 20°C	0.385 kJ/(kg °C)
Coefficient of linear expansion at 20°C - 100°C	16.8 x 10 ⁶ °C ⁻¹
Coefficient of linear expansion at 20°C - 300°C	17.7 x 10 ⁶ °C ⁻¹
Modulus of elasticity E	118 000 N/mm ²
Shear modulus G	44 000 N/mm ²
Thermal conductivity(λ) at 20°C	ca 365 W/(m °C)
Poisson's ratio ν	0.34

All materials shrink or expand with temperature changes. It is necessary to take into account the coefficient of thermal expansion of each material in order to reliably determine the degree of change. This is expressed as the movement which occurs in the material in the event of a 1 °C change in the temperature.

The longitudinal elongation Δl can be described by mean of the formula below:

$$\Delta l = L\alpha\Delta T$$

L = length of the sheet

α = coefficient of thermal expansion
($16,8 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$)

ΔT = difference in temperature

Note:

Thermal movements must be permitted by the joints. If the edges of the sheet are supported, buckling of the sheet occurs.

Note also that the elastic modulus of copper is 118 000 N/mm², so the thermal stresses due to thermal elongation are smaller than with many other materials.

No Eurocode exists for copper structures yet. Stainless and Aluminium codes may be applied. For plate buckling values, the values of cold formed steel (prEN1993-1-3) are widely used (same as for aluminium).

Copper in contact with other metals

Copper is a noble metal, which is why galvanic corrosion does not normally lead to damage to the surface of the copper. As it is a noble metal, copper can, just like other noble metals, cause galvanic corrosion to other, "less noble" metals such as aluminium, zinc and iron. Therefore, building structures should be designed in such a way as to avoid contact – both direct and indirect – between these metals.

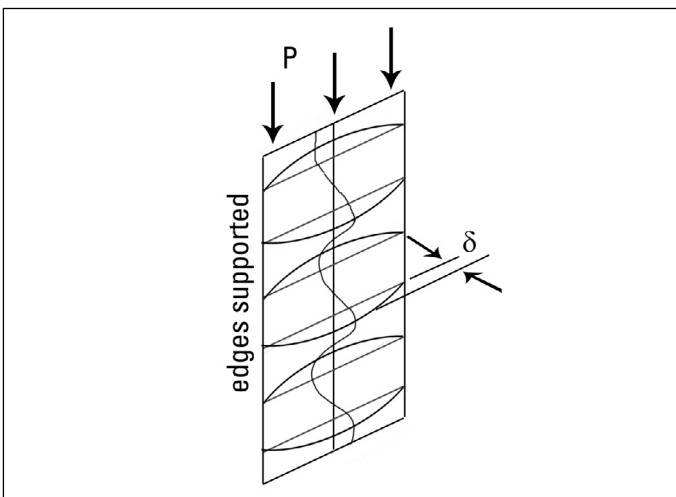
Direct contact between the metals should be prevented by inserting an intermediate layer of non-conductive material, such as a neoprene strip, or by simply leaving a gap.

Arrangement of copper above zinc or steel should be avoided, because the copper ions washed away in the rainwater drain onto the zinc, react with it, and accelerate the zinc's corrosion.

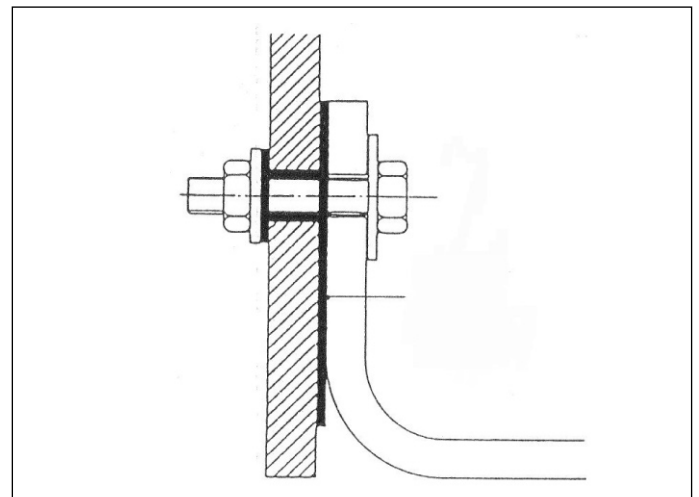
Copper in contact with other building materials

Local damage can occur in the form of erosion corrosion due the shortcomings in detail design. For instance, such corrosion might occur where water and particles of sand constantly leaked out and dripped down onto underlying copper sheeting, thereby abrading the protective surface layer.

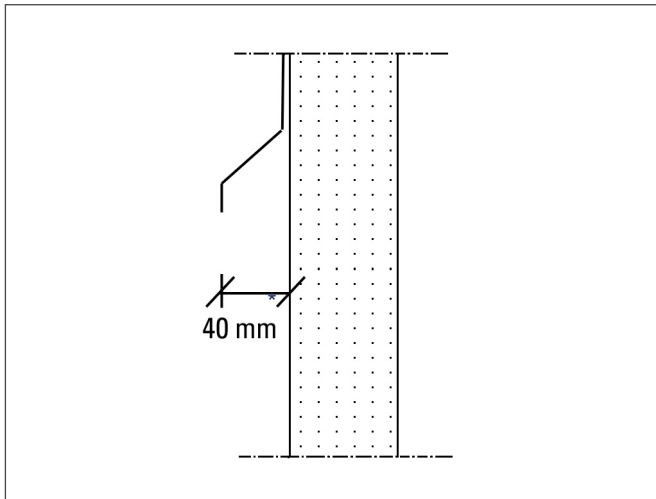
It is inappropriate to place copper sheeting in direct contact with bituminous surfaces because of corrosion. Acid rainwater in concentrated form from a bitum sur-



Buckling of sheet



Non-conductive material in black area



*check local regulations

face, that prevents the development of the protective surface layer on the copper sheeting. In the long term, this leads to discoloration and damage to the copper sheeting.

Façades made of non-water repellent materials such as plaster, sandstone, brick, wood etc. should be protected against rainwater containing copper which could otherwise lead to discoloration. Flashings on plastered walls should be terminated with drip mouldings which run no less than 40mm beyond the finished wall.

Copper does not react to wooden components, but contact with weatherproofed wood should be avoided.

Environmental Issues

Atmospheric-corrosivity categories according to EN ISO 12944-2 and examples of typical environments

Table 5

Corrosivity category	Corrosivity level	Examples of typical environments in a temperate climate, informative	
		Exterior	Interior
C1	Very low	-	Heated buildings with clean atmospheres, e.g. offices, shops, schools and hotels.
C2	Low	Atmospheres with low level of pollution. Mostly rural areas	Unheated buildings where condensation may occur, e. g. depots, sport halls.
C3	Medium	Urban and industrial atmospheres, moderate sulphur dioxide pollution. Coastal areas with low salinity.	Production rooms with high humidity and some air pollution, e. g. food-processing plants, laundries, breweries and dairies.
C4	High	Industrial areas and coastal areas with moderate salinity.	Chemical plants, swimming pools, coastal shipyards.
C5-I	Very high (industrial)	Industrial areas with high humidity and aggressive atmosphere.	Building or areas with almost permanent condensation and with high pollution.
C5-M	Very high (marine)	Coastal and offshore areas with high salinity.	Building or areas with almost permanent condensation and with high pollution.

Fastener material with regard to corrosion environment
(and sheeting material only for information).

Table 6

Classification of environment	Sheet material	Material of fastener					
		Aluminium	Electro galvanized steel. Coat thickness > 7µm	Hot-dip zinc coated steel ^b . Coat thickness >45µm	Stainless steel, case hardened. 1.4006 ^d	Stainless steel, 1.4301 ^d 1.4436 ^d	Monel ^a
C1	A, B, C	X	X	X	X	X	X
	D, E, S	X	X	X	X	X	X
C2	A	X	-	X	X	X	X
	C, D, E	X	-	X	X	X	X
	S	X	-	X	X	X	X
C3	A	X	-	X	-	X	X
	C, E	X	-	X	(X) ^c	(X) ^c	-
	D	X	-	X	-	(X) ^c	X
	S	-	-	X	X	X	X
C4	A	X	-	(X) ^c	-	(X) ^c	-
	D	-	-	X	-	(X) ^c	-
	E	X	-	X	-	(X) ^c	-
	S	-	-	X	-	X	X
C5-I	A	X	-	-	-	(X) ^c	-
	D ^f	-	-	X	-	(X) ^c	-
	S	-	-	-	-	X	-
C5-M	A	X	-	-	-	(X) ^c	-
	D ^f	-	-	X	-	(X) ^c	-
	S	-	-	-	-	X	-

Fastener of steel without coating may be used in corrosion classification class C1.

A =	Aluminium irrespective of surface finish	- =	Type of material not recommended from the corrosion standpoint
B =	Un-coated steel sheet	a =	Refers to rivets only
C =	Hot-dip zinc coated (Z275) or aluzink coated (AZ150) steel sheet	b =	Refers to screws and nuts only
D =	Hot-dip zinc coated steel sheet + coating of paint or plastics	c =	Insulating washer, of material resistant to ageing, between sheeting and fastener
E =	Aluzink coated (AZ185) steel sheet	d =	Stainless steel EN 10 088
S =	Stainless steel	e =	Risk of discoloration
X =	Type of material recommended from the corrosion standpoint	f =	Always check with sheet supplier
(X) =	Type of material recommended from the corrosion standpoint under the specified condition only		

Design Principles for Facades and Roofs

Snow

Snow on the roof can result in major stresses on the roof structures. The volumes of snow that can conceivably stress roofs do, of course, depend on the location of the building (both geographically and locally), but the roofing material (friction) and the design of the roof are also central here.

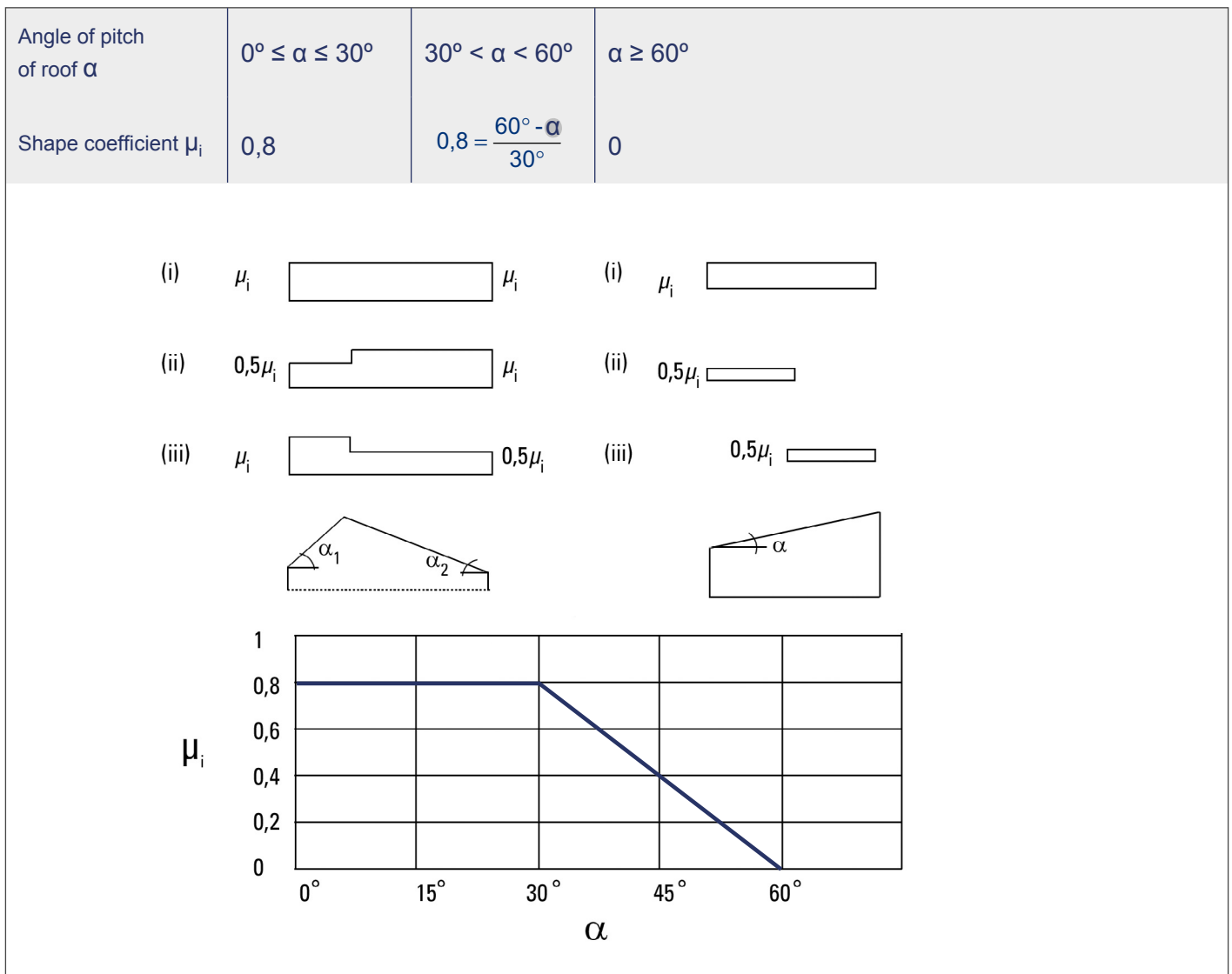
The snow loads on a roof should be determined according to ENV 1993-2-3:

$$s = \mu_i C_e C_t S_k$$

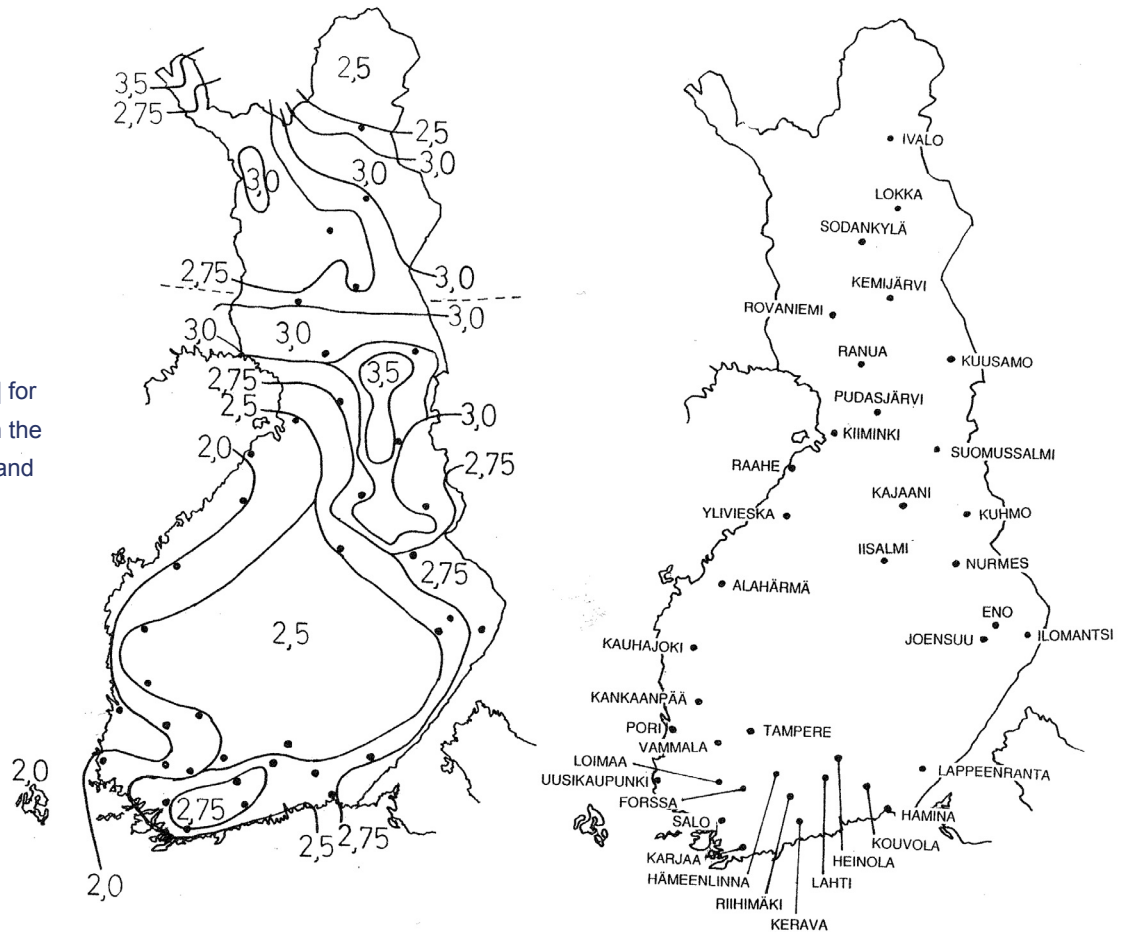
where:

- μ_i = snow load shape coefficient (see figure)
- C_e = snow load exposure coefficient*
- C_t = snow load thermal coefficient*
- S_k = characteristic value of the snow load on the ground [kN/m2]*

*please refer to the regulations applicable in your country.



Characteristic values [kN/m²] for snow loads on the ground in Finland



Wind

Wind loads are the strongest, most frequent hostile factors that affect roofs and façades.

The pressures acting on external surfaces of a structure or a structural component are obtained from the equation on the right.

For details, see your national regulations. In Finland, for example, the reference wind velocity is 21 m/s. The reference wind velocity is the mean velocity at 10 m above flat open country averaged over a period of 10 minutes (typically) with a recurrence interval (return period) of once in 50 years (typically). If the return period is, say, 200 years, then the reference velocity must be 12% larger. If the return period is, say, 5 years, then the reference velocity may be 18% smaller.

$$w_e = q(z) \cdot c_p$$

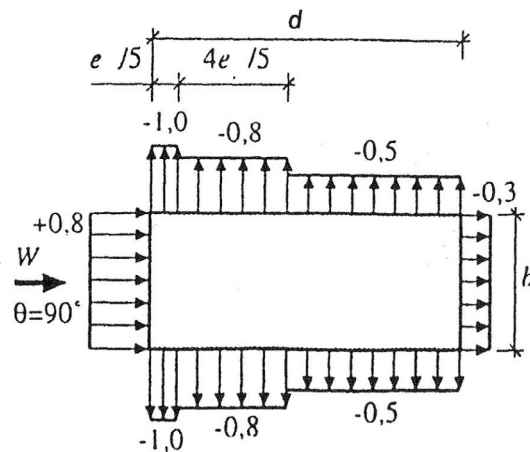
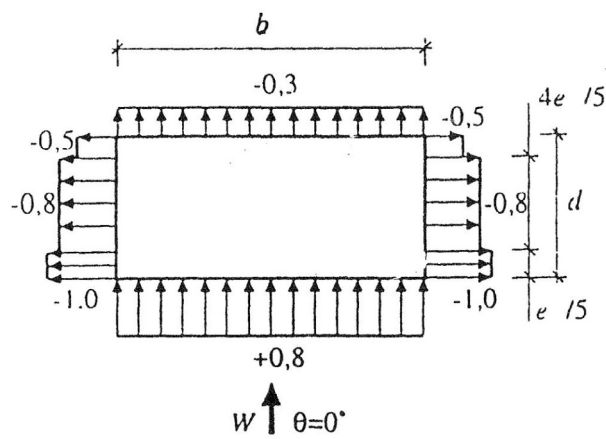
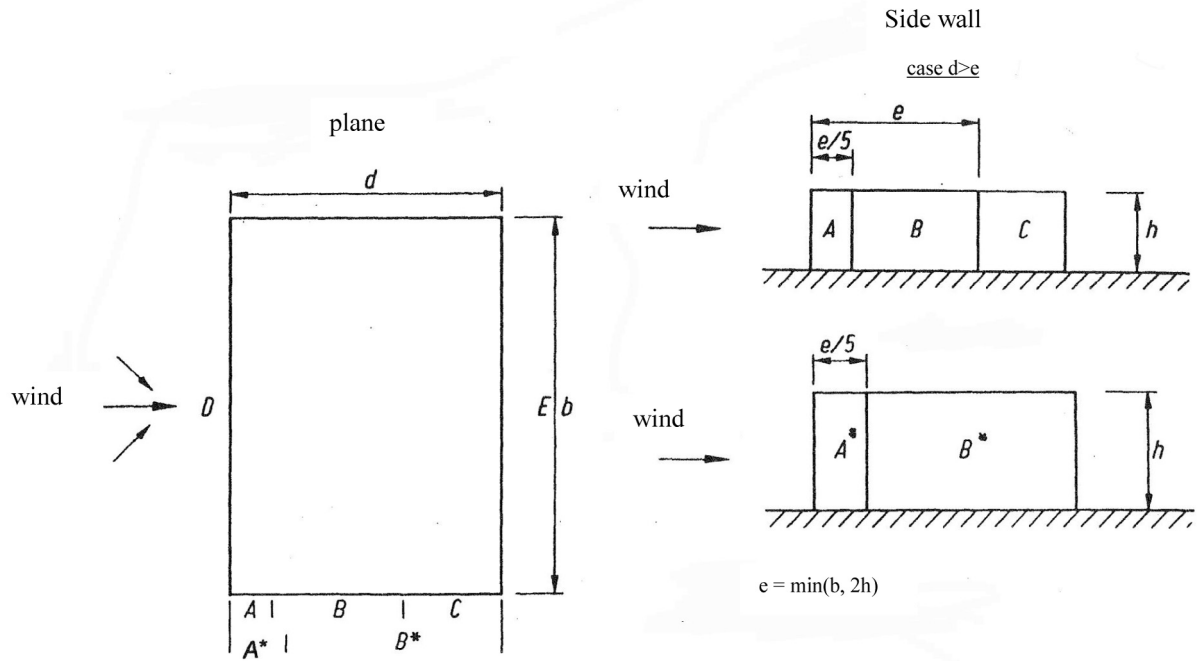
c_p = pressure coefficient (negative or positive). Note that the inner pressure does not typically act on façades or roof covers.

$$q(z) = c_e(z) \cdot q_{ref} = \text{velocity pressure}$$

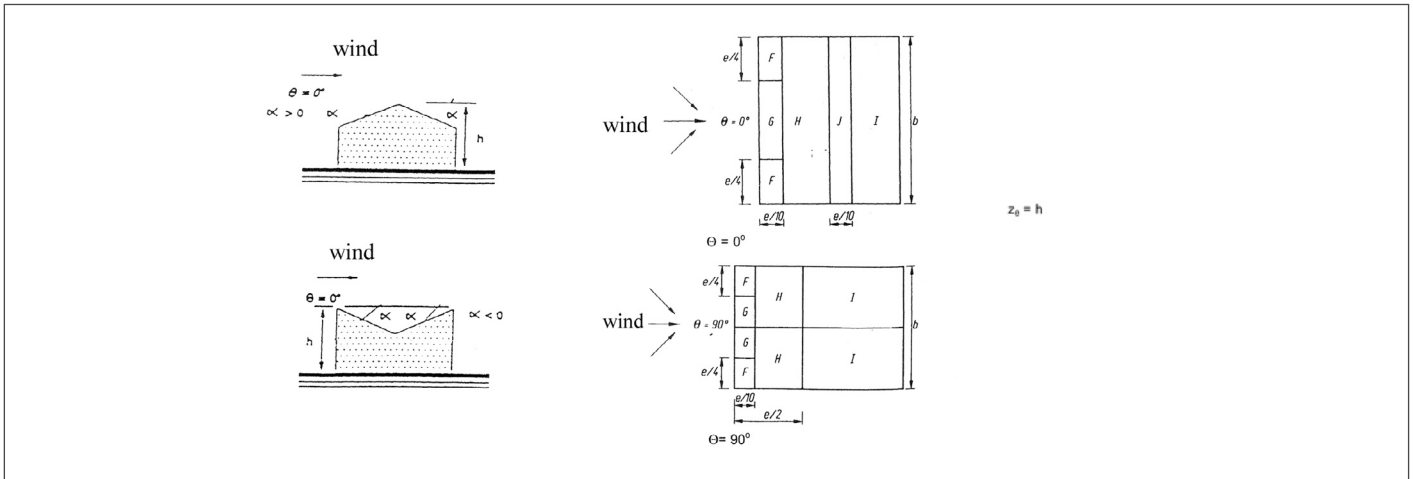
$c_e(z)$ = factor, depending on the height of the building and surrounding conditions

$$q_{ref} = \frac{v_{ref}^2}{1600} \left[\text{kN/m}^2 \right] = \text{reference pressure, } v_{ref} \text{ given in m/s.}$$

Pressure coefficients for walls



Pressure coefficients for roofs



Wind direction 0°

Table 7

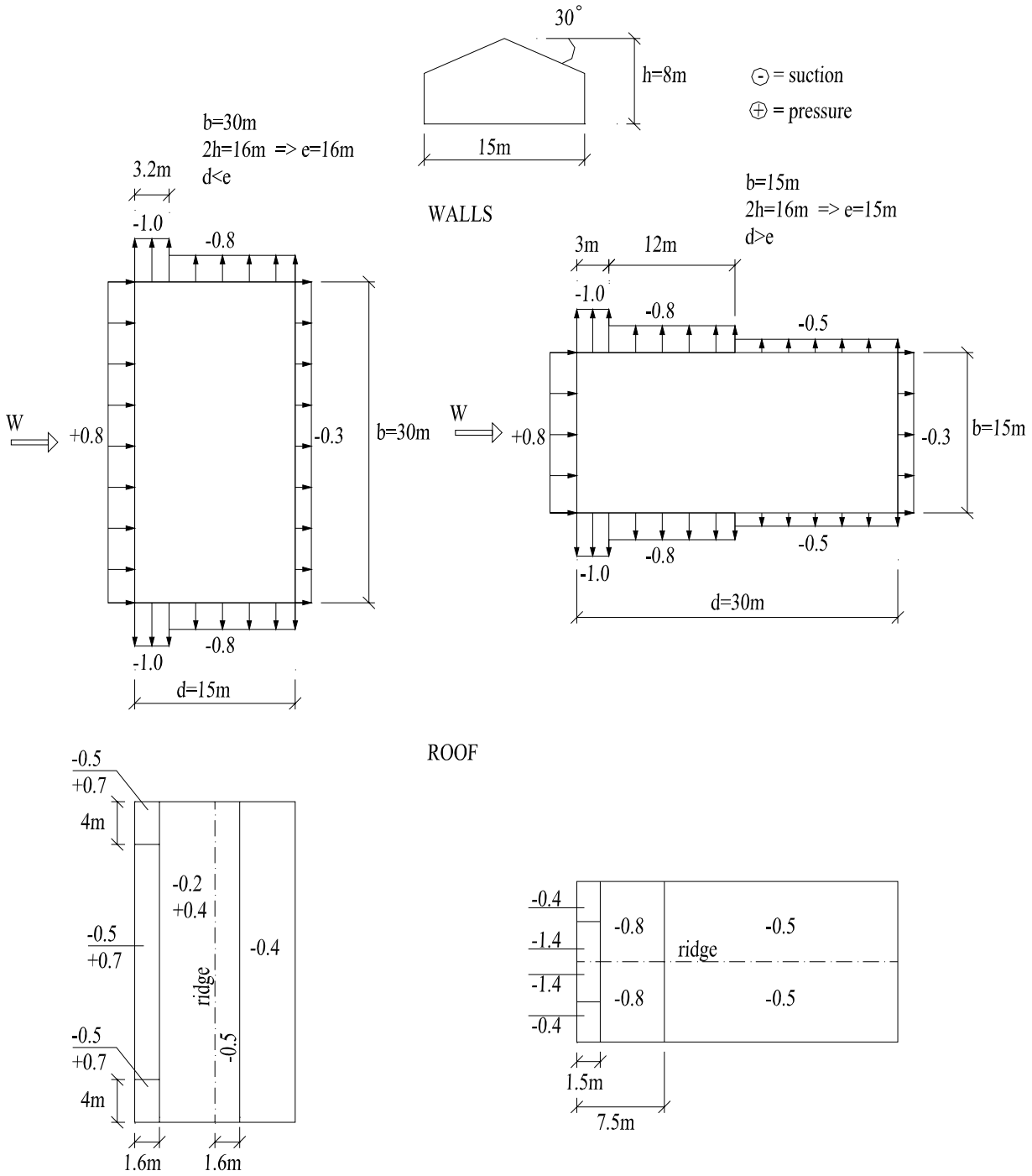
Roof slope α	F		G		H		I		J	
	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$
-45°	-0,6		-0,6		-0,8		-0,7		-1,0	-1,5
-30°	-1,1	-2,0	-0,8	-1,5	-0,8		-0,6		-0,8	-1,4
-15°	-2,5	-2,8	-1,3	-2,0	-0,9	-1,2	-0,5		-0,7	-1,2
-5°	-2,3	-2,5	-1,2	-2,0	-0,8	-1,2	-0,3		-0,3	
5°	-1,7	-2,5	-1,2	-2,0	-0,6	-1,2	-0,3		-0,3	
15°	-0,9	-2,0	-0,8	-1,5	-0,3		-0,4	-1,0	-1,5	
	+0,2		+0,2		+0,2					
30°	-0,5	-1,5	-0,5	-1,5	-0,2		-0,4	-0,5		
	+0,7		+0,7		+0,4					
45°	+0,7		+0,7		+0,6		-0,2		-0,3	
60°	+0,7		+0,7		+0,7		-0,2		-0,3	
75°	+0,8		+0,8		+0,8		-0,2		-0,3	

Wind direction 90°

Table 8

Roof slope α	F		G		H		I	
	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$
-45°	-1,4	-2,0	-1,2	-2,0	-1,0	-1,3	-0,9	-1,2
-30°	-1,5	-2,1	-1,2	-2,0	-1,0	-1,3	-0,9	-1,2
-15°	-1,9	-2,5	-1,2	-2,0	-0,8	-1,2	-0,8	-1,2
-5°	-1,8	-2,5	-1,2	-2,0	-0,7	-1,2	-0,6	-1,2
5°	-1,6	-2,2	-1,3	-2,0	-0,7	-1,2	-0,5	
15°	-1,3	-2,0	-1,3	-2,0	0,6	-1,2	-0,5	
30°	-1,1	-1,5	-1,4	-2,0	-0,8	-1,2	-0,5	
45°	-1,1	-1,5	-1,4	-2,0	-0,9	-1,2	-0,5	
60°	-1,1	-1,5	-1,2	-2,0	-0,8	-1,0	-0,5	
75°	-1,1	-1,5	-1,2	-2,0	-0,8	-1,0	-0,5	

Example of wind pressure coefficients for walls and roof



Temperatures on façade and roof surfaces

Roofs and façades can be subject to great fluctuations in temperature. Specific factors include:

- air temperature
- intensity of solar radiation
- wind speed
- the ability of the surface to dissipate heat
- the thermal resistance and thermal capacity of the roof/exterior wall

When estimating the greatest possible degree of movements for sheet lengths and fasteners, it is the extreme values in respect of temperature differences which are of most interest. However, there is a great risk of fatigue and damage caused by movement, even if the movement is minor but occurs quickly and often.

The highest temperatures on roof surfaces can be calculated using the equivalent temperature:

$$T_e = T_1 + m_u a l$$

where:

- T_e = equivalent outdoor temperature (°C)
 T_1 = air temperature (°C)
 a = roof surface absorption factor (given in the table below)
 l = total solar incident radiation (W/m²)
 m_u = surface coefficient of heat transfer at the surface (m² °C/W)

Example:

On a sunny summer day, the solar incident radiation may be more than 1000 W/m² on a flat metal roof. With an air temperature of +28 °C and a copper sheet surface where $a = 0.9$ and the thermal surface resistance $m_u = 0.05$ m² °C/W, the equivalent temperature is:

$$T_e = 28^{\circ}\text{C} + 0.05 \text{ m}^2 \text{ }^{\circ}\text{C/W} \cdot 0.9 \cdot 1000 \text{ W/m}^2 \\ = 73 \text{ }^{\circ}\text{C}$$

Absorption factors of some of our most common colours and materials

Table 9

Colour, surface	Absorption factor (approx.)	
	New materials	Aged materials
Grey, dark green	a = 0,7	a = 0,7
Light	a = 0,4	a = 0,5
Dark, black	a = 0,9	a = 0,9
White	a = 0,2	a = 0,4
Aluminium sheeting	a = 0,25	a = 0,4 – 0,5
Copper sheeting	a = 0,3 – 0,4	a = 0,9
Metal-coated sheeting	a = 0,25	a = 0,6 – 0,8
Stainless steel sheeting	a = 0,25	a = 0,4

Humidity and rain water

Façades and roofs should be designed following general principles of physics. Tested solutions are generally preferred.



Standards and Regulations

EN 504. Roofing products from metal sheet. Specification for fully supported roofing products of copper sheet.

EN 506. Roofing products from metal sheet. Specification for self-supported products of copper or zinc sheet.

EN 612 Eaves, gutters and rainwater down-pipes of metal sheet. Definitions, classifications and requirements.

EN 1172. Copper and copper alloys. Sheet and strip for building purposes.

EN 1173. Copper and copper alloys. Material condition or temper designation.

EN 1412. Copper and copper alloys. European numbering system.

ENV 1993-1-3. Eurocode 3: Design of steel structures. Part 1-3: General rules. Supplementary rules for cold formed thin gauge members and sheeting.

ENV 1991-1. Basis of design and actions on structures.

EN 10088. (1.4301, AISI 304), Stainless steels, Part 2: Technical delivery conditions for sheet/plate and strip for general purposes.

Design Principles for Façades

Façade behavior

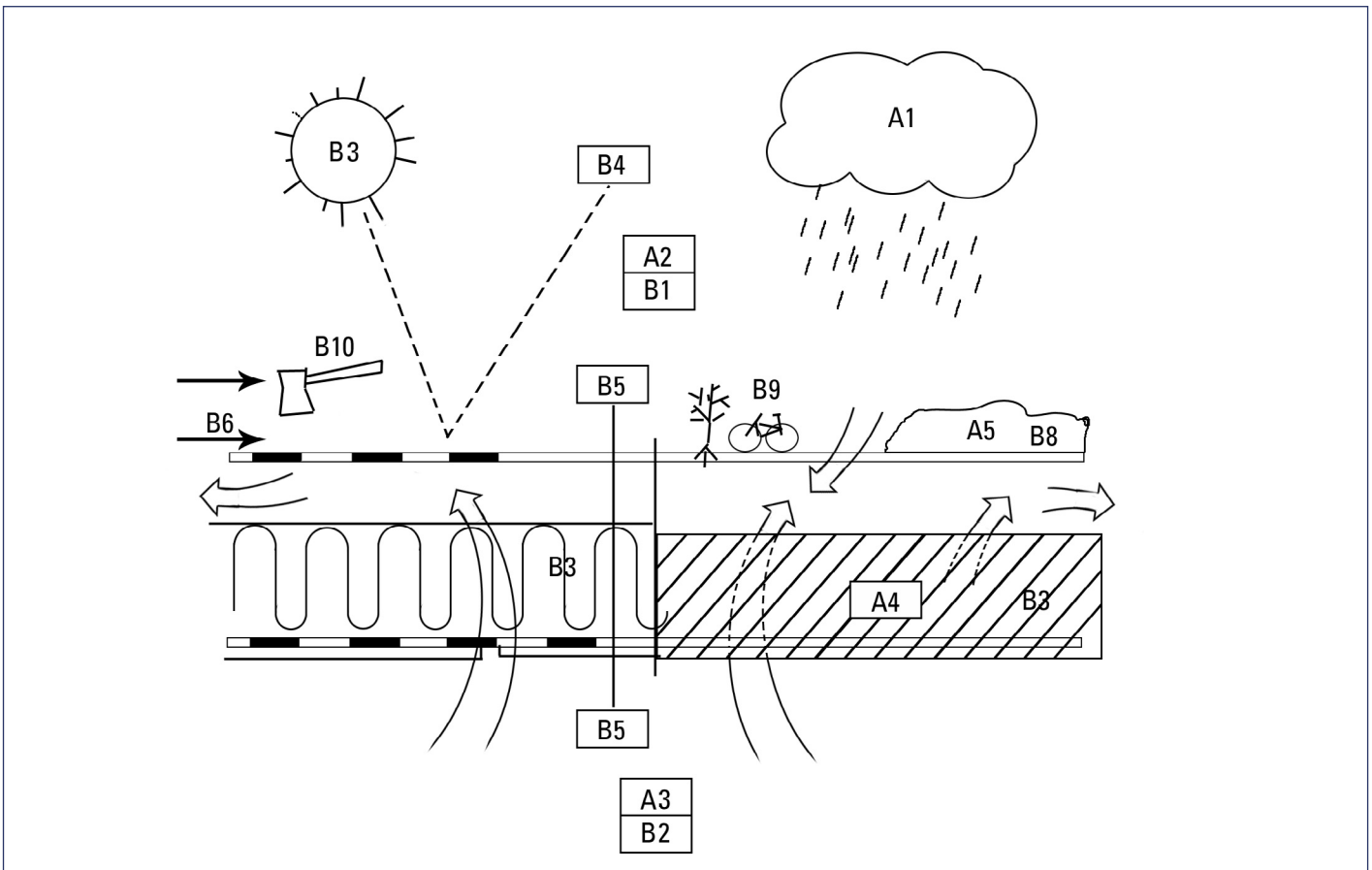
Façades should be designed so that moisture and rain water damage are avoided. The air ventilating gap for all details must be designed with care. The minimum ventilating air gap in façades is 20 mm.

Design Principles for Roofs

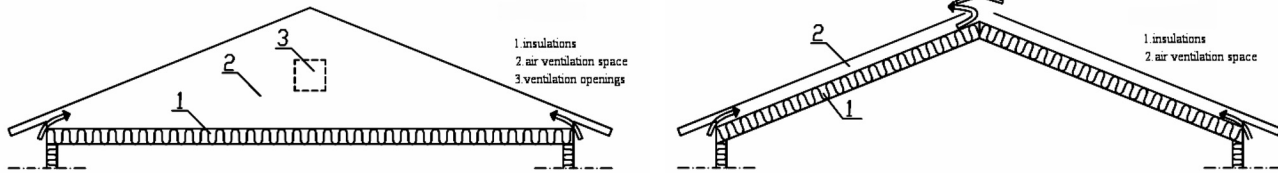
Actions and resistance

The most important task of the roof has always been to protect everything under it – the building, the home and the people.

A. Moisture actions		B. Other actions	
Rain	A1	Outside temperature	B1
Outside moisture	A2	Inside radiating	B2
Inside moisture	A3	Sun radiating	B3
Building damp	A4	Opposite radiating	B4
Snow and ice	A5	Pressure diffencies	B5
		wind	
		temperature	
		ventilation installation	
		Wind	B6
		Actions	B7
		dead weight	
		snow, ice, water	B8
		Biological (plants)	B9
		Mechanical	B10



Roof behavior, air ventilation



Air ventilation of roofing

Table 10

Roof slope	Ventilation space	Inlet	Outlet
<1:20	200 mm	5 ‰	5 ‰
1:20...1:5	100 mm	2 ‰	2,5 ‰
>1:5	75 mm	2 ‰	2,5 ‰

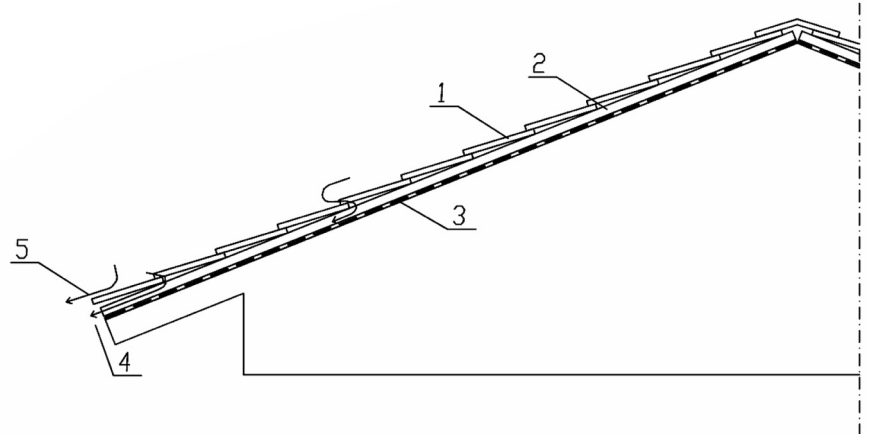
Ventilation dimensioning in accordance with Finnish building regulations.

Check your own regulations.

Water on roofs

Roofing behaviour

1. Overlapped copper roofing
2. Air ventilation and purlins
3. Lower roofing (substrate)
4. Penetration water
5. Rainwater



Slope of roof

The slope of the roof is specified using the terminology on the right.

Copper sheets can be set to a slope of up to 1:12 with special care for penetration water, edges of roof penetrations and tightening the seams. The height of the vertical seam may be increased 5 mm.

Horizontal roofs

0.0° – 0.6° slope, or 1:00 – 1:100

Flat roofs

0.0° – 3.6° slope, or 1:100 – 1:16

Shallow sloping roofs

3.6° – 14.0° slope, or 1:16 – 1:4

Steep roofs

> 14° slope, or > 1:4

Copper sheets with double seams: minimum slope 1:10

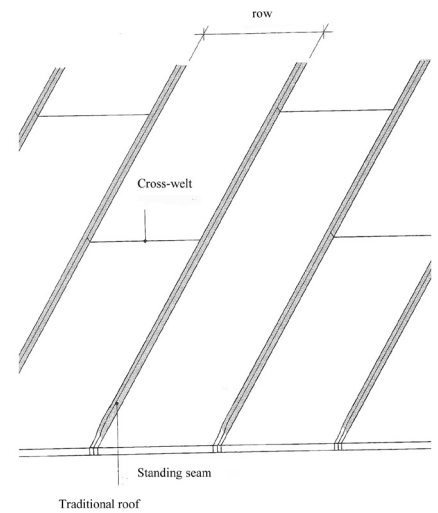
Traditional Roofs

General

Covering roofs with sheet seamed together is known as a traditional roof. The seams are known as standing seams and cross-welts. Traditional sheet covering has a long history and was practised with the beaten sheet metal of the old days. Today, traditional covering is used primarily on buildings where the designer wants to give the roof character, or when original architecture is to be retained and managed.

Strip covering

Today copper is generally manufactured and supplied in strips. These can be cut into panels of any length. The main advantages of long strip are elimination of the many cross-welts required with the traditional methods and considerable scope for prefabrication with on-site machines and mechanised seaming (particularly where anticipated by the roof design).



Substrates

Typically:

- wood or
- plywood
- with bitum belt cover



Seams					
Standing seam					
Batten roll seam					

Fasteners

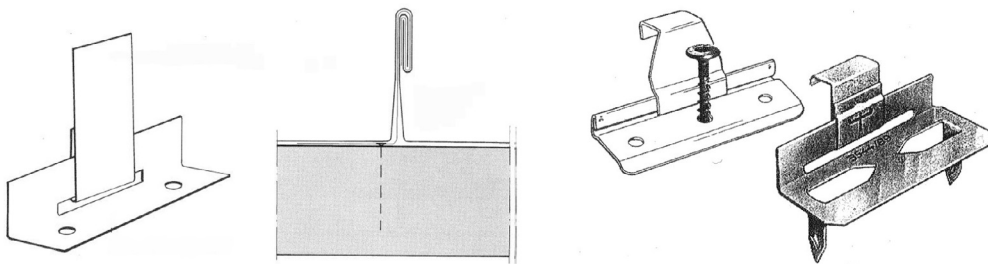
Material

Recommended material:

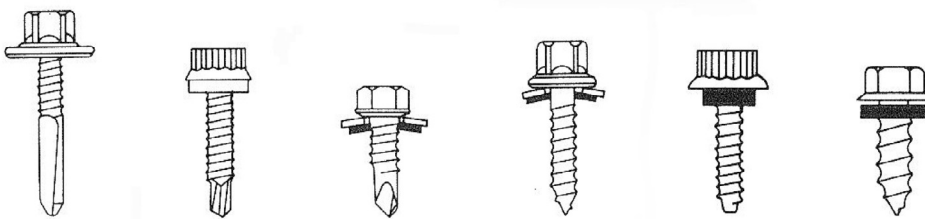
- EN 10088 (1.4301, AISI 304), Stainless steels, Part 2: Technical delivery conditions for sheet/plate and strip for general purposes

Methods of fastening

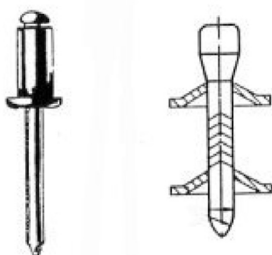
Clips



Self-drilling or tapping screws



Other methods: Shot nails, rivet, bolts, hit anchors



Design sheets

- Material thickness of Cassette
- Calculation of fasteners in tension
- Thermal expansion of copper sheet
- Spacing of timber battens

Background documentation on design sheets

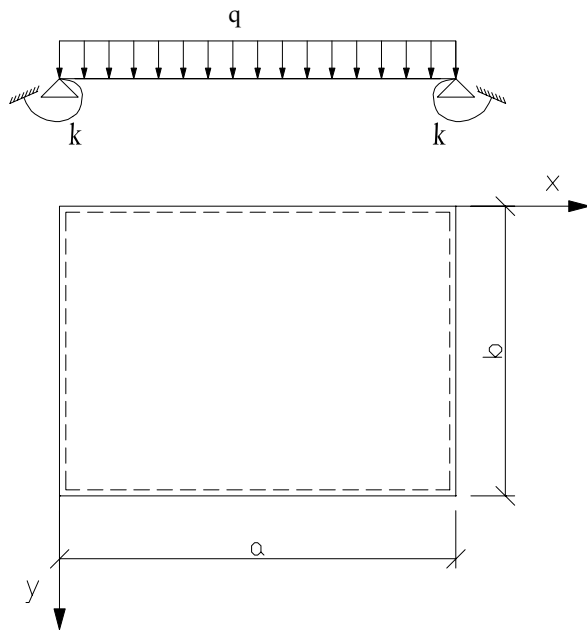
Material thickness of Cassette (t)

The structural analysis of the cassette is done in two phases:

- Definition of the thickness t
- Definition of the height of the cassette H

The thickness (t) of the cassette must be such that the deflection of the cassette is not too much (check your national regulations for the deflection limits) and the copper sheet can resist all the actions (check your national regulations for safety factors). The height (H) of the cassette must be such that the edge of the cassette can resist all actions.

In this design sheet, the uniform load (typically wind) against the cassette is analyzed. The Euro-code system applies.



Uniform load (q , wind). Rectangular ($a \cdot b$) simply supported (hinges) thin plate with rotational springs at all four edges.

Navier's solution for a simply supported thin plate:

Deflection at any point x, y ($m = n = 29$, for every sum):

$$v_q(x, y) = \frac{16q}{\pi^6 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{mn \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 \right]^2}$$

Bending moment M_x :

$$M_x(x, y) = \frac{16q}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left(\frac{m}{a} \right)^2 + \nu \left(\frac{n}{b} \right)^2}{mn \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 \right]^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

Bending moment M_y :

$$M_y(x, y) = \frac{16q}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\nu \left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2}{mn \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 \right]^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (m = 1, 3, 5, \dots) \text{ and } (n = 1, 3, 5, \dots)$$

Bending stiffness:

$$D = \frac{Et^3}{12(1-\nu^2)}$$

where E is the elastic modulus of the copper sheet and ν is the Poisson's ratio of the copper sheet. The thickness t is the design thickness (= nominal thickness-tolerances) of the copper cassette sheet.

Axial stresses:

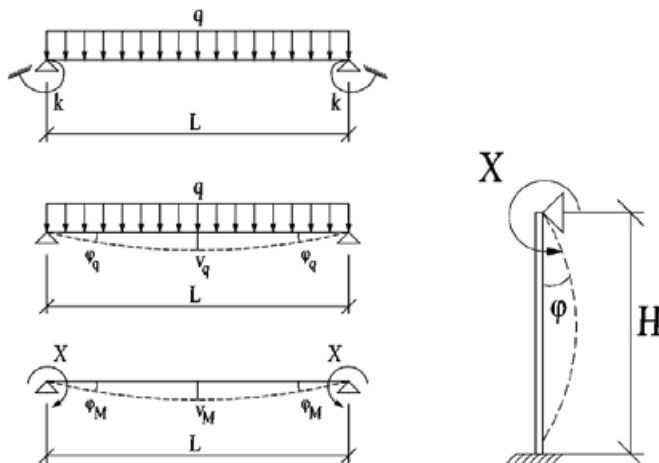
$$\sigma_x = \frac{6}{t^3} M_x$$

These stresses (at maximum, mid-point of the plate) are given as the output of the design sheet. These values must be compared to the proper design values of the copper sheet.

$$\sigma_y = \frac{6}{t^3} M_y$$

Rotational stiffness is based on the configuration of the edge of the cassette.

The rotational stiffness of the edge is taken into account only for the deflections. The rotational stiffness coefficient is derived following the beam theory using the models shown in the following.



$$v = \left[1 - \frac{4}{5 \cdot \left(\frac{H}{2L} + 1 \right)} \right] * v_q$$

where

v_q = Navier's solution given above

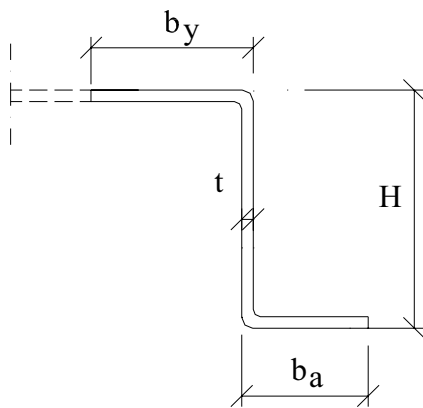
H = height of the cassette

L = min(a,b), a and b are the dimensions of the cassette.

The final deflection is given as the output of the design sheet.

Height of cassette (H)

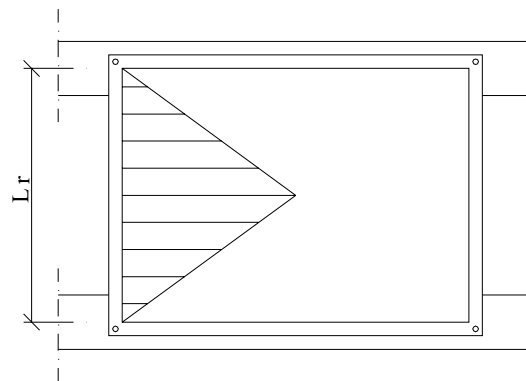
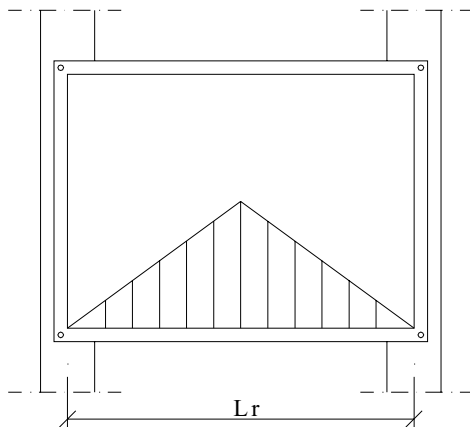
The edge of the cassette is as below:



The lower width (b_a) depends on the cassette and it is given as an input to the design sheet.

Typically, two opposite edges are not supported continuously, as shown in the following figure. For the uplift load (wind suction) the cassette is typically supported only at the connecting points as shown in the figure. The user of the design sheet defines, whether a or b is the critical length for the cassette.

The edge is analysed as a simple supported beam with span L_r and the cross section as stated below. The loading for this beam is triangular loading based on load on the cassette (see figure below).

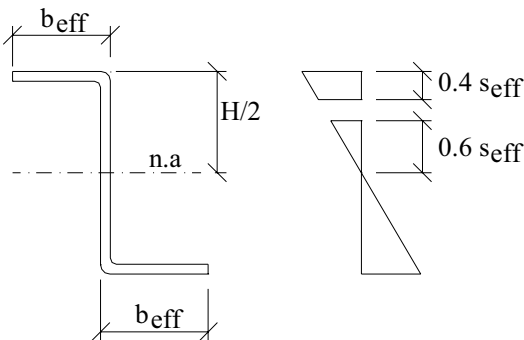
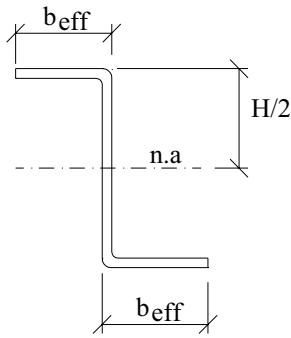


Analysis procedure for determining height H of the cassette:

Calculate the effective width for b_a and b_y ($b/t \leq 50$) using the outstand element buckling value for cold-formed steel sheet buckling (ENV1993-1-3, see later the table, $k_\sigma = 0.43$) and the gross cross section.

Calculate $b_{\text{eff}} = \min(b_a, b_y)$

The cross section of the beam is shown in the figure on page 23.



Both flanges are fully effective.

Calculate the effective parts of the web using double supported compression elements and tables below ($k_\sigma = 4.0$).

The cross section of the beam is shown in the following figure.

Note that in this version the reduction for the web is not performed. The entire web is taken into account for the resistance calculations.

Calculate the effective bending modulus W_{eff} for this cross section.
Check the resistance of the cross section

$$M_{c,Rd} \leq f_y W_{eff} / \gamma_{M1} \quad \text{where } f_y \text{ is the yield stress (or more precisely 0.2\% proof strength) for the copper and } \gamma_{M1} \text{ is the material safety factor (} \gamma_{M1} = 1.1 \text{ is recommended, if not otherwise stated in the national regulations). Note that action } M_{c,Rd} \text{ includes the load safety factor. A value of 1.5 for wind is recommended, if not otherwise stated in the national regulations.}$$

The result (the strength check of the edge) is given as an output of the design sheet.

Local buckling following ENV 1993-1-3

The reduction factor ρ for the plate element width is as follows:

If $\bar{\lambda}_p \leq 0.673$: $\rho = 1.0$
 If $\bar{\lambda}_p > 0.673$: $\rho = (1.0 - \frac{0.22}{\bar{\lambda}_p}) / \bar{\lambda}_p$, where:

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \equiv \frac{b_p}{t} \sqrt{\frac{12(1-\nu^2)f_y}{\pi^2 E k_\sigma}} \quad \text{, where: } b_p \text{ is the nominal width of the plate element under consideration and } f_y \text{ is the yield stress (or, more precisely, 0.2\% proof strength) for the copper sheet.}$$

k_σ is given in tables 11 and 12.

Doubly supported compression elements

Table 11

Stress distribution [compression positive]				Effective width b_{eff}		
				$\psi = +1:$ $b_{eff} = \rho b_p$ $b_{e1} = 0.5 b_{eff}$ $b_{e2} = 0.5 b_{eff}$		
				$0 \leq \psi < +1:$ $b_{eff} = \rho b_p$ $b_{e1} = \frac{2b_{eff}}{5 - \psi}$ $b_{e2} = b_{eff} - b_{e1}$		
				$-1 \leq \psi < 0:$ $b_{eff} = \rho b_c$ $b_{e1} = 0.4 b_{eff}$ $b_{e2} = 0.6 b_{eff}$		
				$\psi < -1:$ $b_{eff} = \rho b_c$ $b_{e1} = 0.4 b_{eff}$ $b_{e2} = 0.6 b_{eff}$		
$\psi = \sigma_2 / \sigma_1$	+1	$+1 > \psi > 0$	0	$0 > \psi > -1$	-1	$-1 > \psi > -3$
Buckling factor k_σ	4.0	$\frac{8.2}{1.05 + \psi}$	7.81	$7.81 - 6.29\psi + 9.78\psi^2$	23.9	$5.98(1 - \psi)^2$

Alternatively, for $+1 \geq \psi \geq -1$:

$$k_\sigma = \frac{16}{\left[(1 + \psi)^2 + 0.112(1 - \psi)^2 \right]^{0.5} + (1 + \psi)}$$

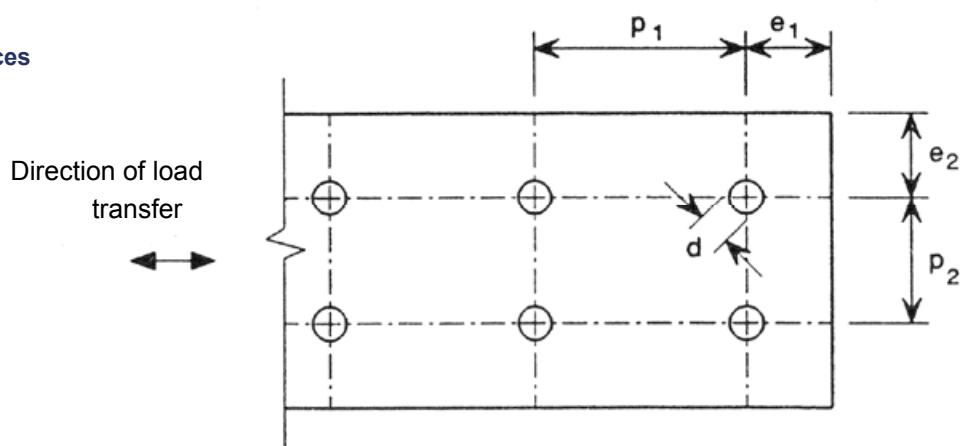
Outstand compression elements

Table 12

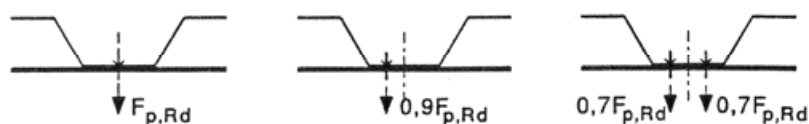
Stress distribution [compression positive]		Effective width b_{eff}				
		$0 [\psi < +1:$ $b_{eff} = \rho b_p$				
		$\psi < 0:$ $b_{eff} = \rho b_c$				
$\psi = \sigma_2 / \sigma_1$ Buckling factor k_σ	+1	0	-1	$+1 \geq \psi \geq -1$ $0.57 - 0.21\psi + 0.07\psi^2$		
		$0 [\psi < +1:$ $b_{eff} = \rho b_p$				
		$\psi < 0:$ $b_{eff} = \rho b_c$				
$\psi = \sigma_2 / \sigma_1$	+1	+1 > ψ > 0	0	$0 > \psi > -1$		-1
Buckling Factor k_σ	0.43	$\frac{0.578}{\psi + 0.34}$	1.70	$1.70 - 5\psi + 17.1\psi^2$		23.8

Calculation of screws in tension following ENV 1993-1-3

Edge distances



Reduction of tension resistance due to screw location in the structure.

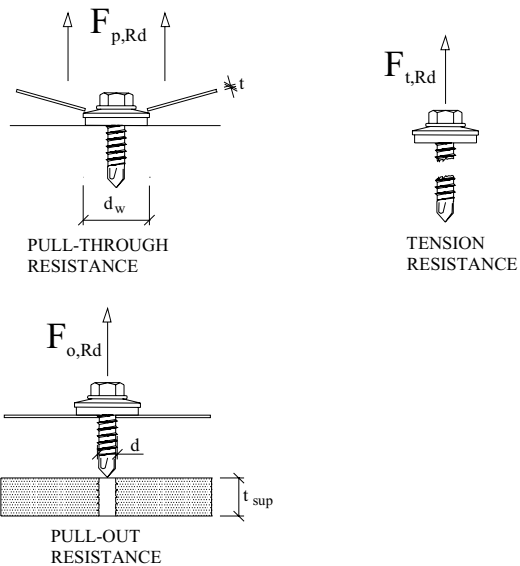


Notations:

d	nominal diameter of a screw
d_w	diameter of a washer or a screw end
e_1	edge distance in force direction
e_2	perpendicular edge distance
f_u	characteristic ultimate tensile strength of copper sheet
$f_{ub,sup}$	characteristic ultimate tensile strength of supporting material
p_1	screw distance in force direction
p_2	screw distance in perpendicular direction
t	copper sheet design thickness (=nominal thickness-tolerances)
t_{sup}	supporting material thickness

Recommended value for the material safety factor of joints
(please, check your national regulations): $\gamma_{M2} = 1.25$

Tensile resistance checks (ultimate limit states)



Screws loaded in tension:

Pull-through resistance: $F_{p,Rd} = 0.5 d_w t f_u / \gamma_{M2}$ (for screws subject to wind loads and combination of wind loads and static loads)

Pull-out resistance: $F_{o,Rd} = 0.65 d t_{sup} f_{u,sup} / \gamma_{M2}$

Tension resistance: Tension resistance $F_{t,Rd}$ to be determined by testing (given by fabricator of the screw).

Conditions: $F_{t,Rd} \geq F_{p,Rd}$ or $F_{t,Rd} \geq F_{o,Rd}$

The required conditions should be fulfilled when deformation capacity of the connections is needed. When these conditions are not fulfilled, it should be determined that the needed deformation capacity will be provided by other parts of the structure.

Range of validity:

Generally: $e1 \geq 3 d$ $p1 \geq 3 d$ $3.0 \text{ mm} \leq d \leq 8.0 \text{ mm}$

$e2 \geq 1.5 d$ $p2 \geq 3 d$

For tension: $0.5 \text{ mm} \leq t \leq 1.5 \text{ mm}$ $f_u [550 \text{ MPa}$

Thermal elongation and buckling of copper sheet

Thermal elongation is

$$\Delta l = L\alpha\Delta T$$

L = Length of the sheet

α = Thermal elongation coefficient for copper

ΔT = Difference of temperatures

Thermal strain is:

$$\varepsilon = \frac{\Delta l}{L} = \alpha\Delta T$$

If thermal strain is prevented, then there is stress:

$$\sigma = E\varepsilon$$

Example: When erecting the sheet the temperature is +20 °C and the length is $L = 15\text{m}$. Suppose the temperature rises to +75 °C, then

$$\Delta l = 15000\text{mm} \cdot 17 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1} \cdot (75^\circ\text{C} - 20^\circ\text{C}) = 14\text{mm}$$

If this is prevented, then there is the axial stress

$$\sigma = 118000 \frac{\text{N}}{\text{mm}^2} \cdot 17 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1} \cdot 55^\circ\text{C} = 110 \frac{\text{N}}{\text{mm}^2}$$

The critical buckling stress is

$$\sigma_{\text{cr}} = \frac{\pi^2 E k_\sigma}{12(1-\nu^2)} \left(\frac{t}{b_p} \right)^2$$

E elastic modulus 118 000 N/mm²

k_σ buckling factor, typically $k_\sigma = 4.0$ (see tables)

ν Poisson's ratio 0.34

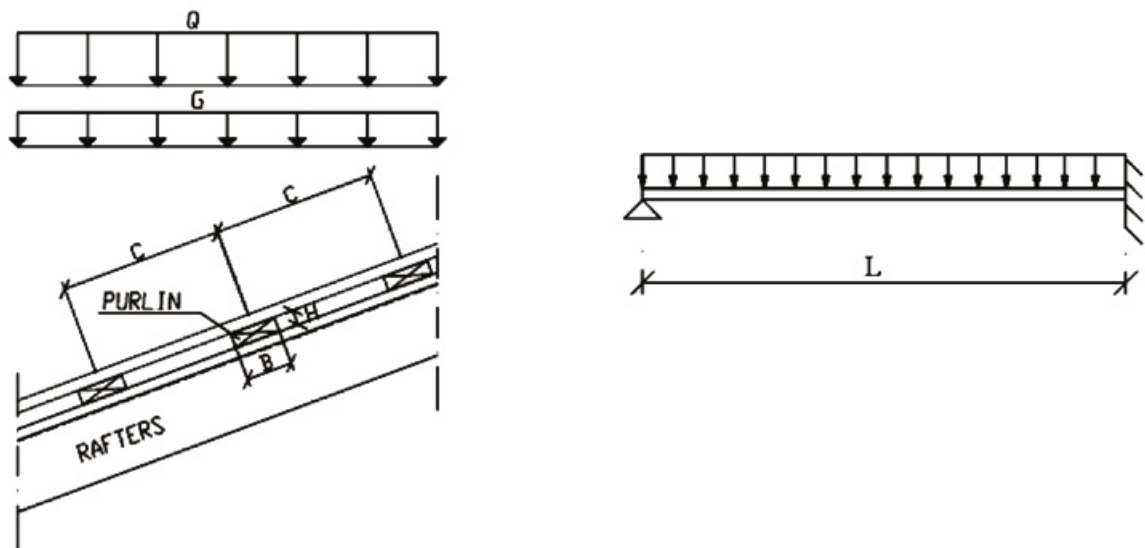
t design thickness of the copper sheet

b_p nominal width of the copper sheet

Buckling is typically avoided when the joints have sufficient deformation capacity.

Spacing of timber battens

The timber battens can be analyzed using beam theory as follows.
Geometrical layout of the case is as shown in the following.



The analysis model for one purlin is shown in the figure, too. As an input the wood material and the loading data must be given. The design is done using Eurocode ENV 1995 (Wooden structures). The relevant equations for the beam are:

Bending moment

$$M = \frac{qL^2}{8}$$

Shear force

$$Q = \frac{5qL}{8}$$

Deflection

$$v = \frac{qL^4}{185EI}$$

Axial stress

$$\sigma = \frac{M}{W}$$

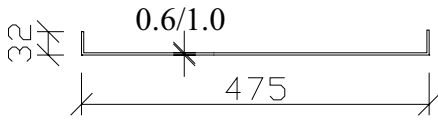
Shear stress

$$\tau = 1.5 \frac{Q}{A}$$

The elastic modulus is following the wood material and environmental items, as are the allowed stresses. The cross-sectional values (I , W , A) are calculated according to the purlin dimensions.

Allowed load for prefabricated roof system

The prefabricated roofing parts which have the following idealized cross-sections, can be analyzed as follows. The similar calculations can be made for other products, too.



The result for this profile is given in table 13. The table has been calculated for copper grade DHP-R240 using the deflection limit $L/50$ (L is the smaller dimension of the sheet), the design thickness 0.58mm (nominal thickness 0.6 mm). The deflection limit is always the most critical for uniform load. The given loads in the table are characteristic loads without safety factor. The material safety factor used in the calculations is 1.1 and the load factor at the ultimate limit state is 1.5. The same values can be calculated for nominal thickness 1.0 mm (design thickness 0.98 mm). The result is given in table 14.

Note that the allowed point load is not much. Thus if there is space between purlins, as there usually is with profiled sheets, then walking on the roof is not allowed without separate walkways.

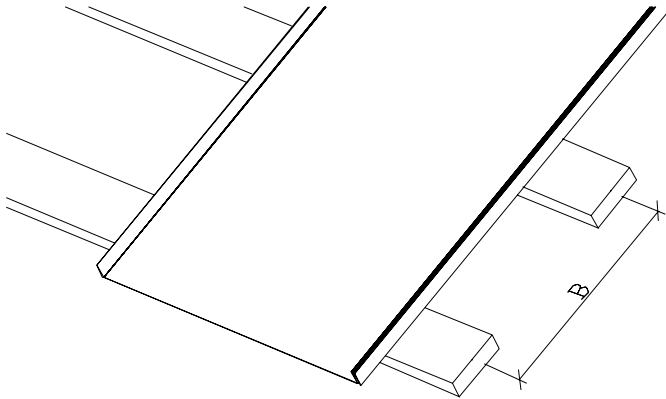
Table 13

Purlin spacing B [mm]	Allowed uniform load ($t=0.58$ mm) [kN/m ²]	Allowed point load ($t=0.58$ mm) [kN]
150	2.50	0.18
200	1.05	0.14
250	0.55	0.12
300	0.37	0.11
350	0.26	0.10

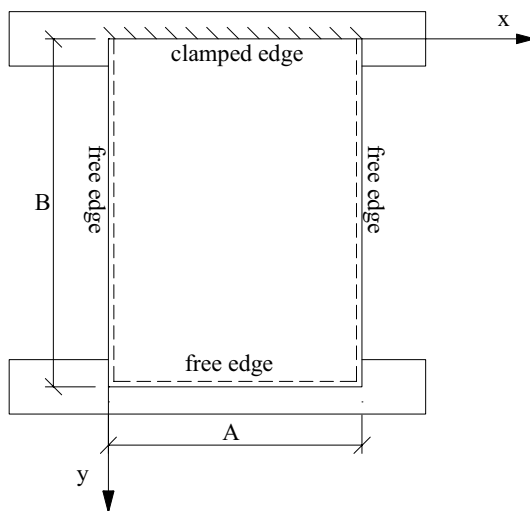
Table 14

Purlin spacing B [mm]	Allowed uniform load ($t=0.98$ mm) [kN/m ²]	Allowed point load ($t=0.98$ mm) [kN]
150	8.0	0.53
200	4.8	0.41
250	2.7	0.35
300	1.7	0.31
350	1.2	0.29

The prefabricated roof system considered looks as follows:



The analysis model for this part is the thin plate supported with hinges at three edges and rigidly supported at one edge is shown in the following figure.



The deflection of the plate is (p_0 = uniform load, $n = 1, 3, 5, \dots$, x, y coordinates) as Levy's solution

$$w(x, y) = \sum_{n=1,3,5,\dots}^{\infty} [(A_n + B_n \alpha_n y) \cosh \alpha_n y + (C_n + D_n \alpha_n y) \sinh \alpha_n y + Q_n] \sin \alpha_n x$$

where

$$Q_n = \frac{4p_0 A^4}{n^5 \pi^5 D}$$

$$A_n = -\frac{4p_0 A^4}{n^5 \pi^5 D}$$

$$B_n = Q_n \frac{\cosh^2 \beta_n - \cosh \beta_n - \frac{1}{2} \beta_n \sinh \beta_n}{\beta_n - \sinh \beta_n \cosh \beta_n}$$

$$C_n = -B_n$$

$$D_n = Q_n \frac{-2 \sinh \beta_n \cosh \beta_n + \sinh \beta_n + \beta_n \cosh \beta_n}{2(\beta_n - \sinh \beta_n \cosh \beta_n)}$$

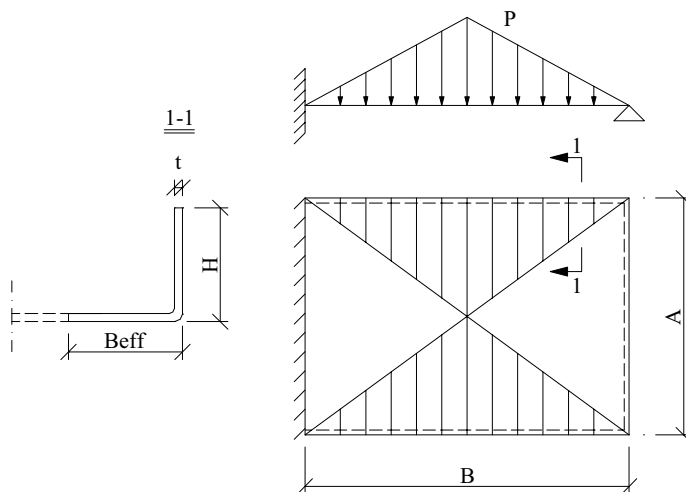
$$\beta_n = \alpha_n B, \quad \alpha_n = \frac{n\pi}{A}$$

The bending stiffness, where E is the elastic modulus, ν is the Poisson's ratio and t is the design thickness of the sheet, is

$$D = \frac{Et^3}{12(1-\nu^2)}$$

The bending moments and shear forces can be calculated by deriving the deflection formula given above.

The loading of the edge beam is triangular, as shown in the following figure.



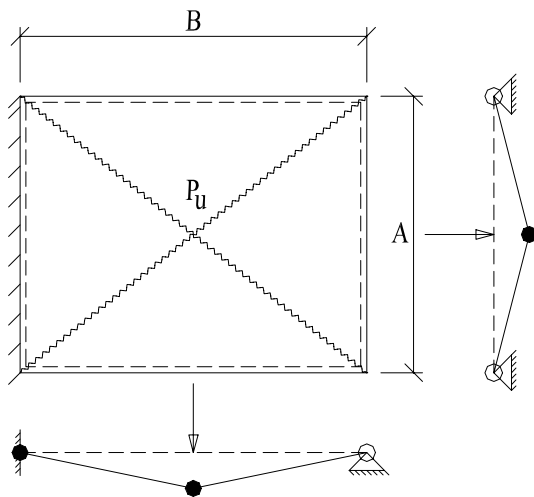
The idealized cross-section of the edge beam is also shown in the figure. The width of the lower part (B_p) can be safely used the value $50 \cdot t$, so the result is valid for uplift load.

The bending moment for the edge beam is

$$M = -\frac{5p_0AB^2}{128}$$

The edge beam resistance can be checked using this value for the bending moment and the bending modulus for the idealized cross-section.

The resistance for the point load can be calculated using the yield line theory. The yield lines look as follows (black circles are plastic hinges, yield lines).



Applying the virtual work equation for the yield line pattern the ultimate point load is

$$P_U = 4m_p \left(\frac{B}{A} + \frac{A}{B} \right) + \frac{2A}{B} m_p$$

where the plastic moment m_p is

$$m_p = w_p \cdot \frac{f_y}{\gamma_{M0}} = \frac{t^2}{4} \cdot \frac{f_y}{\gamma_{M0}}$$

and f_y is the yield stress (or more precisely 0.2% proof strength) and t is the design thickness (=nominal thickness-tolerances).

The ultimate point load on the other hand, is

$$P_U = \gamma_k \cdot P$$

where γ_k and γ_{M0} are the load safety factor and the material safety factor, correspondingly, and P is the characteristic point load.

If national regulations do not state otherwise, it is recommended to use $\gamma_k = 1.5$ and $\gamma_{M0} = 1.1$.